MÉTHODOLOGIE DE SYNTHÈSE

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COCORAM AVANCEMENT

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System structure

- Global system composed by a filter network and the darlington equivalent of the load.

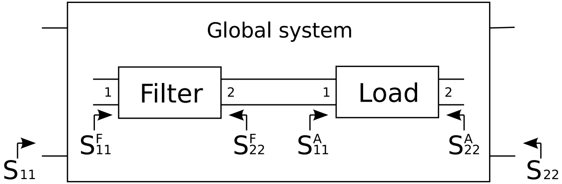

Figure 1: Global system.

Global view of the matching problem.

- Design of the global system.
- A part of the system corresponding to the load is fixed.
- Interpolation conditions to ensure the extraction of the load.

Interpolation condition to extract the load [Bode-Fano]

Considerations (Load of degree 1)

$$S^A = \frac{1}{q^A}\begin{pmatrix} p^{A*} & -r^{A*} \\ r^A & p^A \end{pmatrix} \qquad deg(p^A) = deg(q^A) = 1 \qquad deg(r^A) = 0$$

$$S = \frac{1}{q}\begin{pmatrix} p^{*} & -r^{*} \\ r & p \end{pmatrix} \qquad deg(p) = deg(q) = N \qquad deg(r) < N$$

Extraction of the load

- Condition on the derivative at the zeros of r^A to ensure the extraction.
$$S_{22}^{'}(\omega_t) \leq S_{22}^{A'}(\omega_t) \qquad \forall\ \omega_t : r^A(\omega_t) = 0$$
- Load of degree 1 and $r^A = cte \longrightarrow$ only one condition at infinity.

Statement of the problem

$$\textit{Find}: \qquad \min_{p} \max_{\omega} \left(|S_{22}(\omega)|\right) \qquad \omega \in [-1, 1]$$

$$\textit{Subject to}: \qquad S_{22}^{'}(1/\omega) \leq S_{22}^{A'}(1/\omega) \qquad \omega = 0$$

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Using the Hilbert transform.

Some statements

- Load of degree 1: best S_{22} is minimum phase.
- $|S_{22}(1/\omega)|_{\omega=0} = 1$, then $S'_{22}(1/\omega)\big|_{\omega=0}$ represents the *angular derivative*.
- $ln\,(S_{22}(\omega))$; $\omega \in \mathbb{C}^-$ analytic. Phase of S_{22} is computed from the *Hilbert transform*:

$$\Phi_{22}(\omega) = \frac{1}{\pi} \oint_{\mathbb{R}} \frac{ln|S_{22}(\tau)|}{\tau - \omega} d\tau + C$$

Derivative of Φ_{22} from the *Hilbert transform* (Bode formula).

$$\frac{d}{d\omega}\Phi_{22}\left(\frac{1}{\omega}\right)\bigg|_{\omega=0} = -\frac{1}{\pi}\int_{\mathbb{R}} ln|S_{22}(\tau)|d\tau = \frac{1}{2\pi}\int_{\mathbb{R}} ln\left(1 + \left|\frac{r}{p(\tau)}\right|\right) d\tau$$

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Using the Hilbert transform.

The problem remains...

$$\begin{aligned} Find: \quad & \min_{p} \max_{\omega} \left(|S_{22}(\omega)|\right) \qquad \omega \in [-1,1] \\ Subject\ to: \quad & -\frac{1}{\pi}\int_{\mathbb{R}} ln|S_{22}(\tau)|d\tau \leq K \end{aligned}$$

- Classical filter synthesis.
- The maximum surface of $log(S_{22})$ is constrained.

Remarks.

- $-log|S_{22}(\omega)| \longrightarrow$ Greater when $S_{22}(\omega) \to 0$.
- Matching points on the real axis $\longrightarrow$ strong effect on $\int_{\mathbb{R}} ln|S_{22}(\tau)|d\tau$.
- Intelligent use of the available surface $\longrightarrow$ flat response within [-1,1].
- The matching points are shifted into the complex plane.
- Minimum phase system $\longrightarrow \omega_k's \in \mathbb{C}^+$ (antianalycity domain).

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Hard bound for the reflection level

Optimum reflection function when $N \to \infty$

- $g_{opt}(\omega)$ between all functions with bounded analytic continuation in $\mathbb{C}^+$.
- Surface of $log|g_{opt}|$ fixed.

$$-\frac{1}{\pi}\int_{\mathbb{R}} ln|g_{opt}(\tau)|d\tau = K$$

- g_{opt} equal to a constant $L \leq 1$ within the passband and equal to 1 outside.

$$g_{opt}(\omega) = L \qquad |\omega| \leq 1$$
$$g_{opt}(\omega) = 1 \qquad |\omega| > 1$$

Best reflection level (L)

$$-\frac{1}{\pi}\int_{\mathbb{R}} ln|g_{opt}(\tau)|d\tau = -\frac{1}{\pi}\int_{-1}^{1} ln(L)d\tau = -\frac{2\ ln(L)}{\pi} = K \longrightarrow L = e^{-\frac{\pi K}{2}}$$

Reflection level in dB linear with K

$$RL(dB) = -10\pi K/ln(10)$$

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Optimum filter of finite degree

Matching problem for a load of degree 1

- Defining $P(\omega) = |p(\omega)/r|^2$. The original problem is stated as follows:

$$\textit{Find}: \quad L = \min_{P} \max_{\omega} \; P(\omega) \qquad \omega \in [-1,1] \qquad P(\omega) \geq 0 \;\forall\omega$$

$$\textit{Subject to}: \quad K \geq \frac{1}{\pi}\int_{\mathbb{R}} ln\left(1 + P(\omega)^{-1}\right) d\tau \qquad K \geq 0$$

- The integral can be expressed as:

$$\int_{\mathbb{R}} ln\left(1 + P(\omega)^{-1}\right) d\tau = \int_{\mathbb{R}} \frac{\omega P'(\omega)}{P^2(\omega) + P(\omega)} d\tau$$

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Results

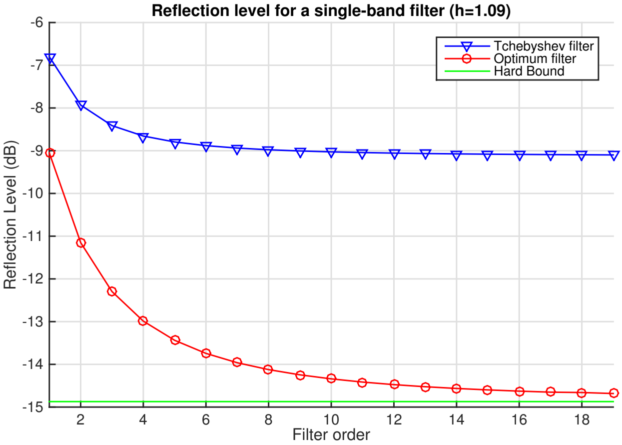

Figure 2: Reflection level with respect to the filter order (N).

Optimal filter for the first band.

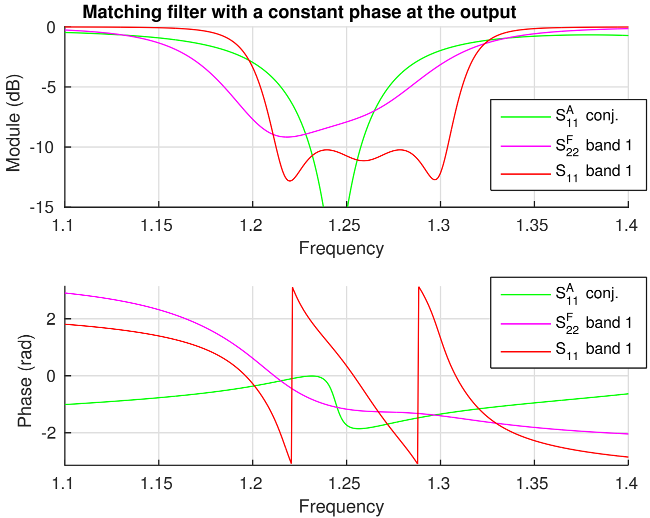

Figure 3: Matching filter and result with a constant phase at the filter output.

Consideration for manufacturing.

Not possible to implement a constant phase shift

- Phase implemented with a transmission line. Phase shift linear in frequency.

$$S_{22}^{F}(shifted) = S_{22}^{F} \cdot e^{-2j\beta(f)l}$$

- Fix-point algorithm to implement the linear phase shifting starting with the constant phase previously obtained.
- Slightly increment in the reflection level.

Aditional input phase in the simulated model.

- Small phase shifting due to access ports of 5 mm.
- Phase shifting due to the input couplings.
- The additional phase shifting was extracted with *Presto* and included in the rational model.

$$S_{22}^{F}(real) = S_{22}^{F}(rational) \cdot e^{-j(Af+B)}$$

Real model of the matching filter.

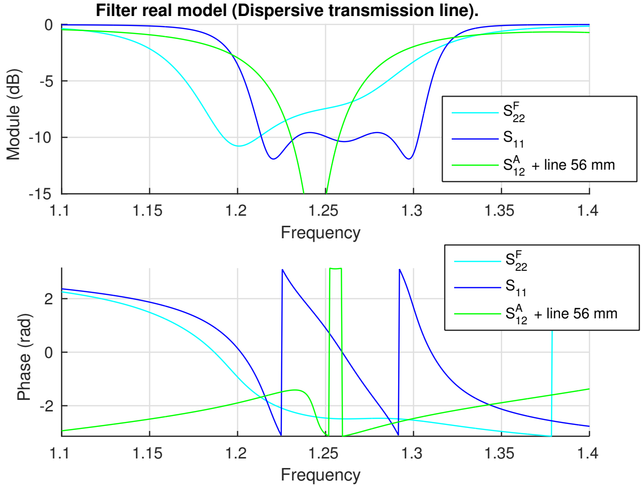

Figure 4: Real model of the matching filter including the input and output phase and a dispersive transmission line at the filter output.

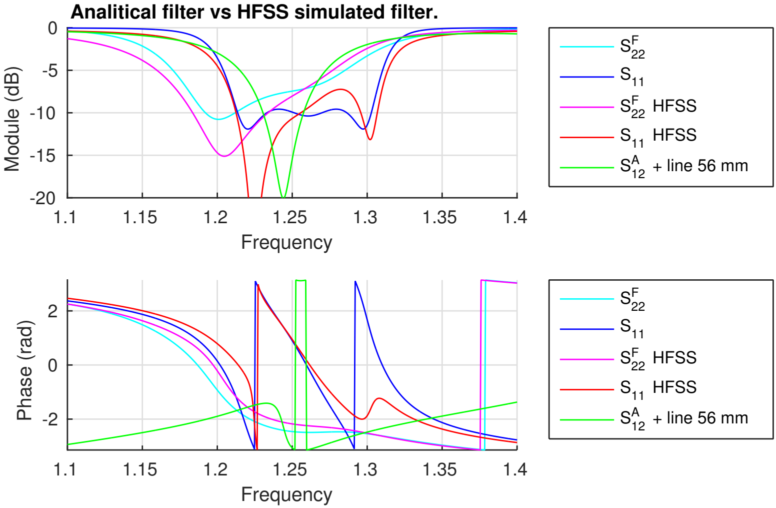

Figure 5: Theoretical filter and HFSS simulation.

Full-wave simulation.

- Target coupling matrix

$$M^T = j \begin{bmatrix} 0 & 1.133 & 0 & 0 \\ 1.133 & 0.065 & 1.144 & 0 \\ 0 & 1.144 & 0.980 & 0.951 \\ 0 & 0 & 0.951 & 0 \end{bmatrix}$$

- Obtained matrix (extracted from the filter)

$$M^O = \begin{bmatrix} 0.006 & 0 & 0 & 0 \\ 0 & 0.140 & -0.004 & 0 \\ 0 & -0.004 & 0.139 & 0 \\ 0 & 0 & 0 & 0.007 \end{bmatrix} + j \begin{bmatrix} 0 & 1.100 & 0.134 & 0.011 \\ 1.100 & 0.043 & 1.033 & 0.118 \\ 0.134 & 1.033 & 0.916 & 0.966 \\ 0.011 & 0.118 & 0.966 & 0 \end{bmatrix}$$

Remarks

- Higer losses at the left edge of the passband where $|S_{22}^F|$ is small.
- Unmatch at the right half of the passband.
- Slightly tuning errors.

Expected result after fine tuning.

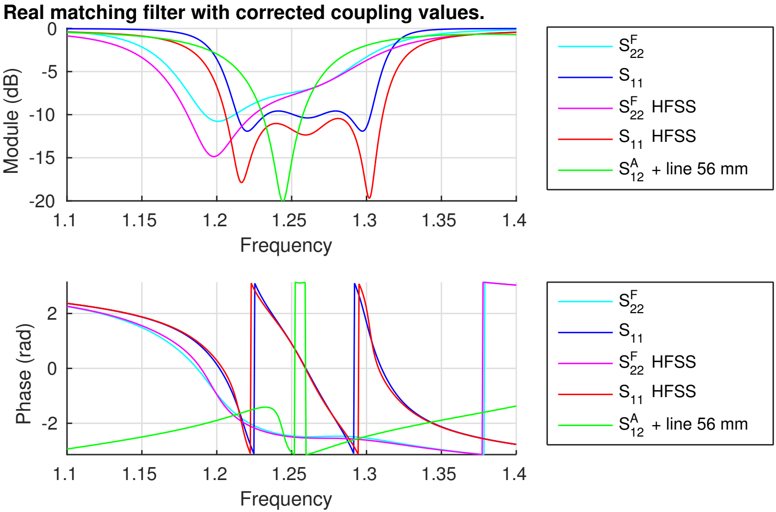

Figure 6: Matching result replacing the imaginary part of M^O by the ideal one.

Conclusions

Filter response to optimize matching.

- The classical *Tchebyshev filter* is not optimum in term of matching.
- The optimal solution can be computed (convex problem).

Design of the microstrip filter.

- The filter tuning should be more precise at the points where the modulus of S_{22}^{F} is higher. (Smaller pseudohyperbolic distance.).
- The filter losses are not critical since they are stronger where the modulus of the S_{22}^{F} is small.

Dual-band matching filter.

Alternatives

- Four-order filter with 2 transmission zeros (high losses at the band edges).
- Five-order inline filter (require broadband tuning).
- Two-order filter for each band with an input-output diplexer.

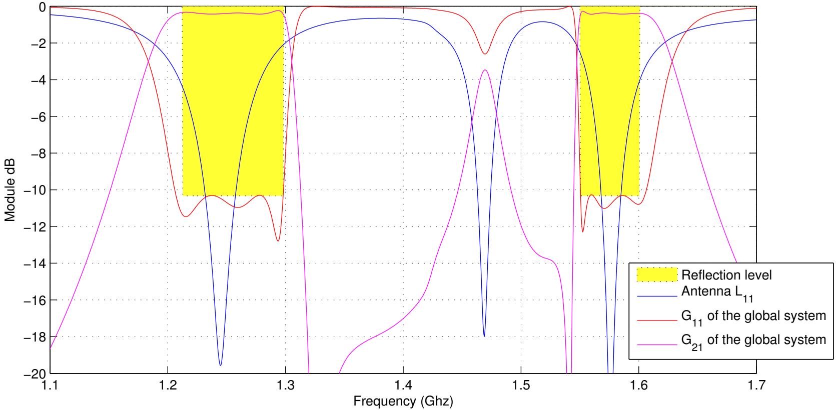

Figure 7: Four-order filter with 2 transmission zeros.

Dual-band matching filter. Alternatives.

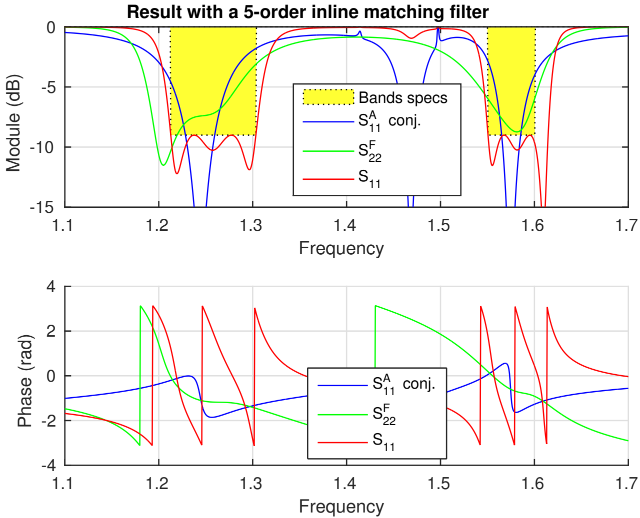

Figure 8: Five-order inline filter.

Dual-band matching filter. Alternatives.

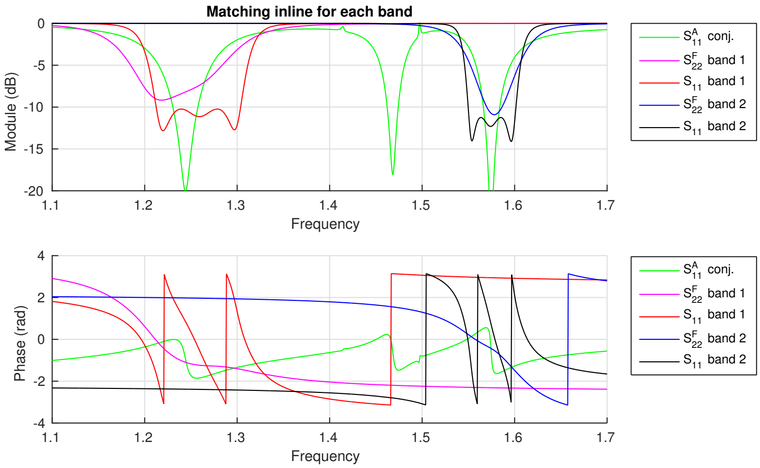

Figure 9: Two-order filter for each band. Derivatives at infinity: $h_1 = 1.09$ $h_2 = 1.14$. Necessity of a diplexer.

Dual-band matching filter. Alternatives.

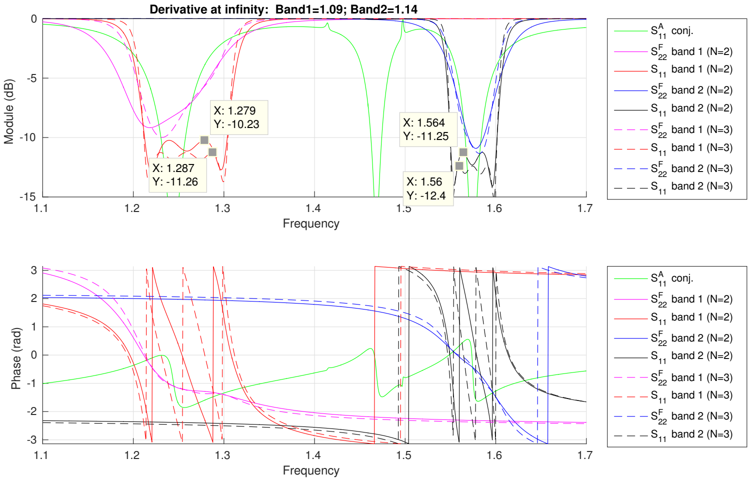

Figure 10: Two-order filter for each band. Comparison N=2 vs N=3.

Dual-band matching filter. Alternatives.

Comparison N=2 vs N=3.

Filter of degree 2.

- Reflection level of -10 dB.
- Dual band filter of degree 4.

Filter of degree 3.

- Reflection level of -11 dB (improvement of 1dB).
- Dual band filter of degree 6.
- Extra losses.