ANR COCORAM
Matching Filter

David Martínez Martínez
Fabien Seyfert
Martine Olivi
Laurent Baratchart

INRIA, Sophia-Antipolis, France

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System structure

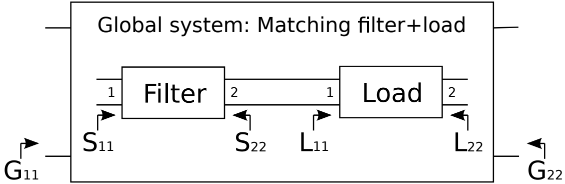

Figure 1: Filter chain.

$$G_{22} = \frac{n}{e} \qquad G_{21} = \frac{m}{e} \qquad ee^* = nn^* + mm^*$$

$$S_{22} = \frac{p}{q} \qquad S_{21} = \frac{r}{q} \qquad qq^* = pp^* + rr^*$$

$$L_{22} = \frac{\varphi}{\psi} \qquad L_{21} = \frac{\rho}{\psi} \qquad \psi\psi^* = \varphi\varphi^* + \rho\rho^*$$

Two equivalent matching problems

Problem 1 (Little Problem)

- *Classical matching problem.*
- *Minimize the pseudo-hyperbolic distance between S_{22} and L_{11}^*.*

$$Find: \min_{p,q} \max_{\omega} \left| \frac{S_{22}(\omega) - L_{11}^*(\omega)}{1 - S_{22}(\omega) L_{11}(\omega)} \right| \qquad \omega \in I \tag{1}$$

Problem 2 (Big Problem)

- *Design the global system G.*
- *Constraints over G_{22} to be able to dechain the antenna afterwards.*

$$Find: \min_{n,e} \max_{\omega} (|G_{22}|) \qquad \omega \in I \tag{2}$$

$$G_{22}(\omega_{\rho_i}) = L_{22}(\omega_{\rho_i}) \tag{3}$$

$$G_{22}'(\omega_{\rho_i}) = L_{22}'(\omega_{\rho_i}) \qquad \forall \omega_{\rho_i} : L_{21}(\omega_{\rho_i}) = 0 \tag{4}$$

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Point-Wise solution of the little problem

Definition of a matching point

$$\text{Matching point } \equiv \omega_k \in \mathbb{R} : \left| \frac{S_{22}(\omega_k) - L_{11}^*(\omega_k)}{1 - S_{22}(\omega_k) L_{11}(\omega_k)} \right| = 0 \qquad (5)$$

Interpolation problem

$$\text{Find:} \quad \frac{p}{q}(\omega_k) = \gamma_k = L_{11}^*(\omega_k) \quad |\gamma_k| < 1 \quad k \in [1, N] \qquad (6)$$

$$\text{Subject to:} \quad qq^* = pp^* + rr^* \quad q \text{ stable}$$

Theorem 1

There exist an unique function $S_{22} = \frac{p}{q}$ stable of degree at most $N - 1$ that satisfies (6) in N different real points ω_k's.

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Point-Wise solution of the little problem

$$deg(L) = 1 \qquad deg(S) = 4 \qquad deg(G) = 5$$

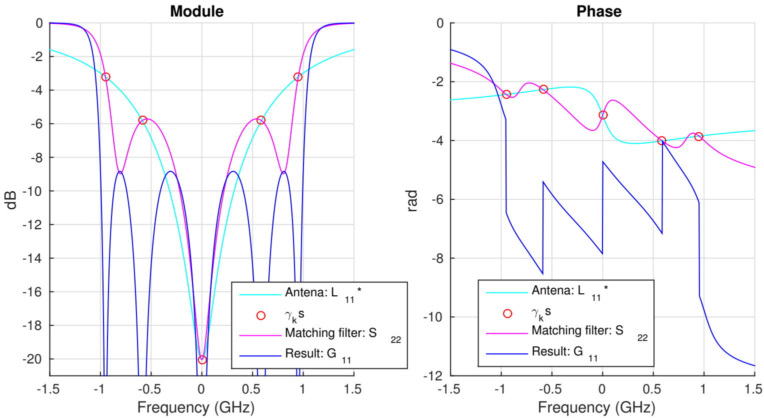

Figure 2: Result of the Point-Wise matching problem.

Obtaining bounds for the reflection level

Solve problem 2 for an antenna of degree 1.

Considerations

- All matching points $\omega_k' s$ (reflection zeros of G) on the real axis.
- Transmission zero of L_{21} at infinity ($\omega_\rho = \infty$).
- $L_{22}'(\omega_\rho) = jh$.
- The system G will be a Tchebyshev filter.

Best result.

$$RL(dB) = -10 \cdot log_{10}\left[1 + sinh^2\left(N \cdot arcsinh\left(h \cdot sin\left(\frac{\pi}{2N}\right)\right)\right)\right]$$

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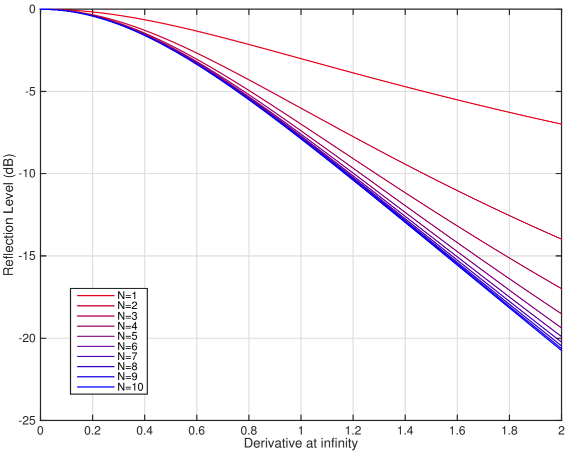

Figure 3: Best reflection level with respect to the derivative (h).

Obtaining bounds for the reflection level

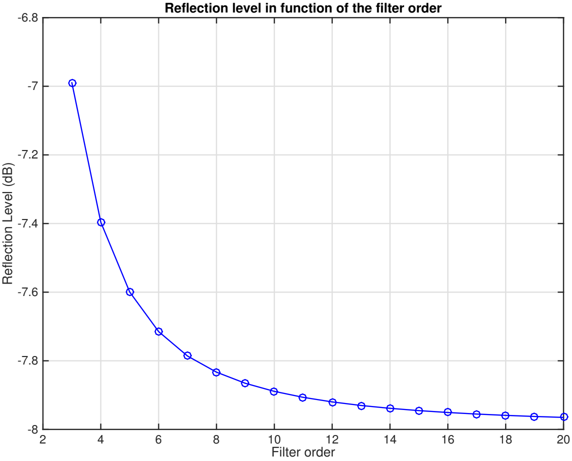

Figure 4: Evolution of the reflection level with the degree of G (h=1).

Bounds with matching points in the complex plane

Considerations

- L of degree 1 and transmission zero of L_{21} at infinity ($\omega_\rho = \infty$).
- $L'_{22}(\omega_\rho) = jh$.
- The reflection and transmission of G equioscillate in the passband.

Equioscillating filter of degree N

$$|G_{22}|^2 = \frac{T_N^2+\alpha^2}{T_N^2+\beta^2} \qquad |G_{21}|^2 = \frac{\beta^2-\alpha^2}{T_N^2+\beta^2} \qquad 0 < \alpha < \beta$$

T_N being the Tchebyshev polynomial of degree N.

Best result

$$RL(dB) = 10 \cdot log_{10} \min_{\alpha} \left(\frac{1+\alpha^2}{1+\beta^2} \right) \tag{7}$$

$$\beta = \sinh \left(N \cdot arcsinh \left(sinh \left(\frac{1}{N} arcsinh\left(\alpha\right) \right) + h \cdot sin\left(\frac{\pi}{2N}\right) \right) \right) \tag{8}$$

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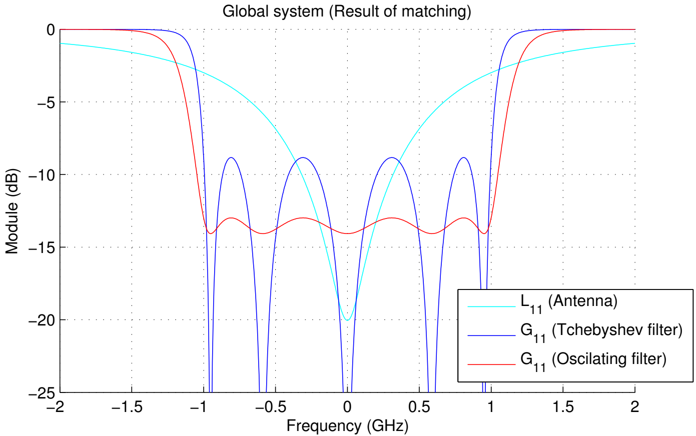

Figure 5: Result of matching with an equioscillating filter.

Bounds Tchebyshev filter vs Equioscillating filter

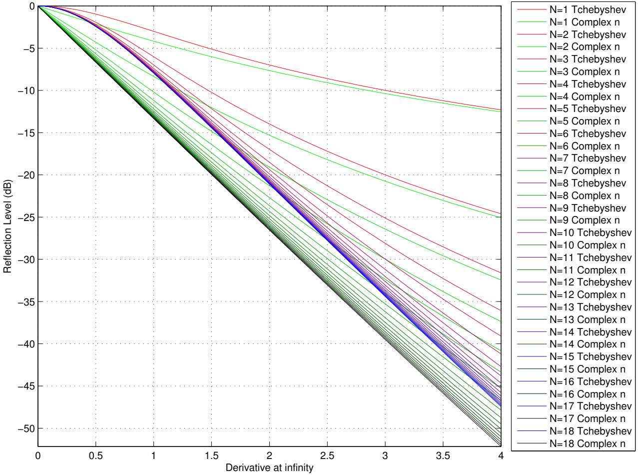

Figure 6: From red to blue: Tchebyshev; From green to black: equioscilating.

Optimum filter by solving the little problem

Considerations

- $S_{22} = \frac{p}{q}$; $L_{11} = \frac{-\varphi^*}{\psi}$.

Problem 1 (Minimize the pseudohyperbolic distance)

Find:

$$\min_{p,r} \max_{\omega} |G_{11}| = \min_{p,r} \max_{\omega} \left| \frac{\frac{p}{q}(\omega) - L_{11}^*(\omega)}{1 - \frac{p}{q}(\omega) L_{11}(\omega)} \right| \qquad \omega \in I \qquad (9)$$

Subject to:

$$qq^* = pp^* + rr^*$$

Optimum filter by solving the little problem

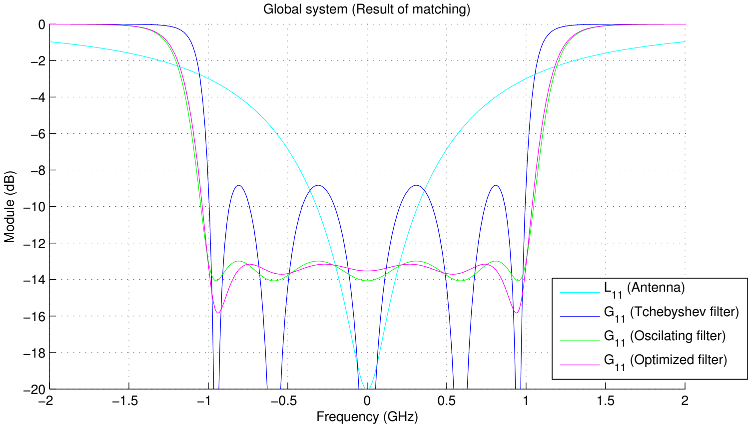

Figure 7: Result obtained for problem 1 (Optimum filter).

Optimum filter by solving the little problem

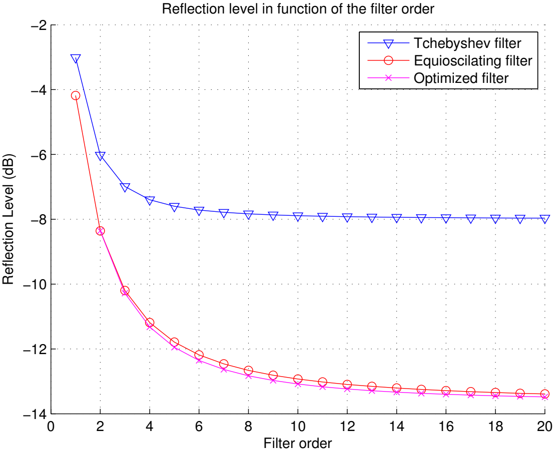

Figure 8: Reflection level of G_{11} with a load of derivative $h = 1$.

Best antenna in term of matching?

Definition for degree 1

- $L_{22} = \frac{\omega - A^*}{\omega - Z}$; $A = a_R + ja_I$; $Z = Z(A)$.
- $L_{21} = \rho$.

Derivative at infinity

$$\frac{\partial L_{22}}{\partial \omega}\left(\frac{1}{\omega}\right)\bigg|_{\omega=0} = j\left(\sqrt{a_I^2 + \rho^2} + a_I\right) = jh$$

- Best matching when $h \to \infty$.

Behaviour of h.

- h does not depend on a_R.
- $h \to \infty$, then $a_I \to \infty$.

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Best antenna in term of matching?

Considerations for the input impedance

- $Z_{in} = \frac{jK}{\omega - Q}$; $K < 0$; $Q = Q_R + jQ_I$, $Q_I > 0$.

Derivative at infinity

$$\left.\frac{\partial L_{22}}{\partial \omega}\left(\frac{1}{\omega}\right)\right|_{\omega=0} = jh = j2Q_I \quad (10)$$

$$\left.\mathrm{Im}\frac{\partial \log(Z_{in})}{\partial \omega}\right|_{\omega=0} = \frac{-1}{Q_I} = \frac{-2}{h} \quad (11)$$

Behaviour of h.

- h only depends on Q_I (Imaginary part of the pole).
- $h \to \infty$, then *derivative of phase*$(Z_{in}) \to 0$.

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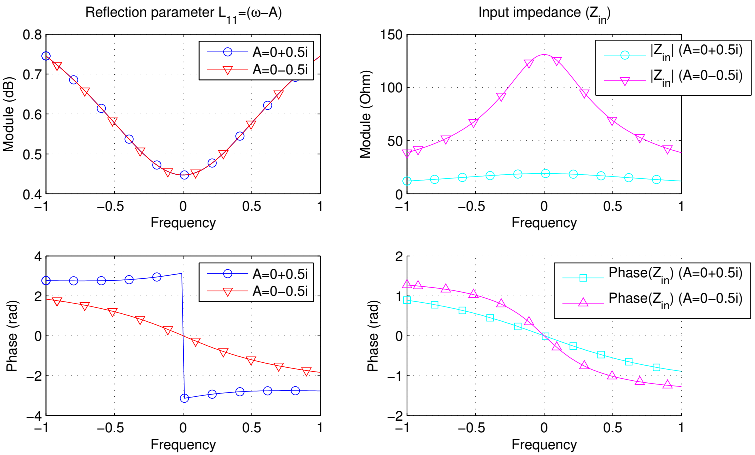
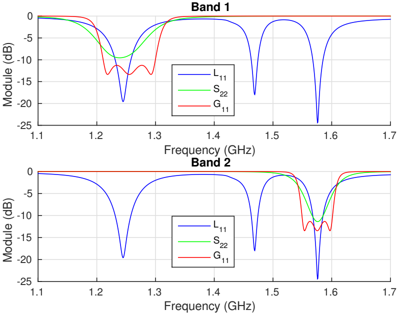


Figure 9: Reflection parameter (L_{11}) and input impedance (Z_{in}) for a load with the reflection zero in the upper half-plane ($A = 0.5j$) and in the lower half-plane ($A = -0.5j$).

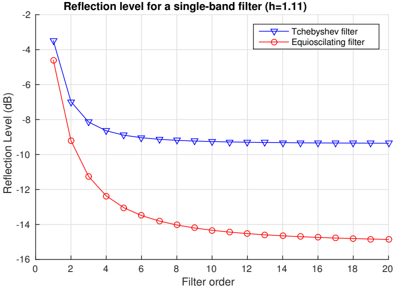
Bounds for a single-band matching filter

Problems to build a dual-band filter.

- Not possible with an inline filter.
- Different constant phase at infinity.



(a) Result for N=3.



(b) Bound for h=1.11 .

Figure 10: Bound of the reflection level for a single-band filter.

Rational approximation of the dual-band antenna

- Rational approximation of degree 3.
- All transmission zeros at infinity.

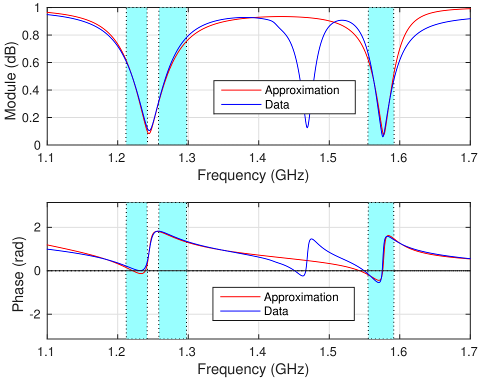


Figure 11: Rational approximation with a load of degree 3.

Dual-band matching solving the little problem.

- Computation: Point-wise matching.

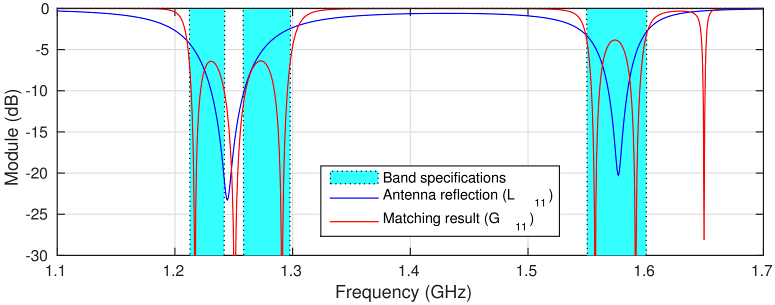


Figure 12: Point-wise solution of the little problem for the dual-band antenna.

Dual-band matching solving the little problem.

- Computation: Point-wise matching + minmax optimization.

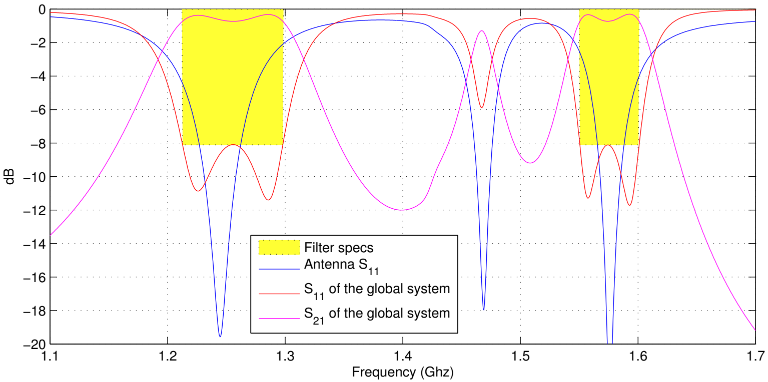


Figure 13: Result of the minmax optimization with the dual-band antenna.

Using the transmission zeros for matching.

k complex conditions: need of k complex parameters.

- Two finite transmission zeros to add one complex parameter (No proof).
- One more adjustable matching point (M+2).

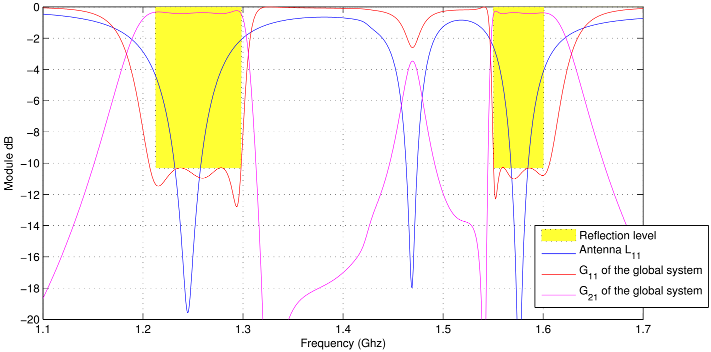


Figure 14: Result of the minmax optimization adding two transmission zeros.

Matching filter

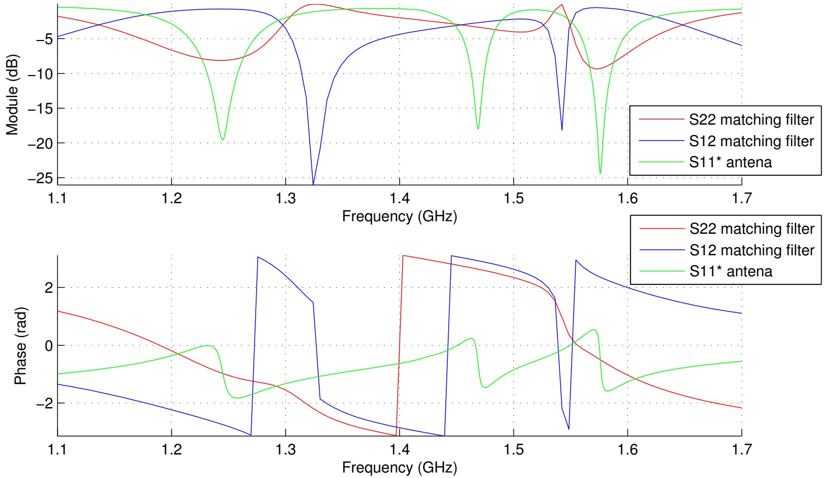


Figure 15: Matching filter.

Matching filter

Polynomial representation

$$p(\omega) = \boldsymbol{e^{2.8607j}} (\omega^4 + (-5.6267 + 0.0433i)\omega^3 + (11.8299 - 0.1885i)\omega^2 + (-11.0143 + 0.2739i)\omega + 3.8321 - 0.1326i) \quad (12)$$

$$q(\omega) = \omega^4 + (-5.6267 - 0.2652i)\omega^3 + (11.7956 + 1.1304i)\omega^2 + (-10.9152 - 1.6009i)\omega + 3.7607 + 0.7532i \quad (13)$$

$$r(\omega) = 0.0416\omega^2 + -0.1193\omega + 0.0851 \quad (14)$$

Coupling matrix

$$M = \begin{bmatrix} 0 & -0.6648 & 0.5512 & -0.1024 & 0.1978 & 0 \\ -0.6648 & 1.1741 & 0 & 0 & 0 & 0.4524 \\ 0.5512 & 0 & -0.9988 & 0 & 0 & 0.4568 \\ -0.1024 & 0 & 0 & -0.7165 & 0 & 0.1942 \\ 0.1978 & 0 & 0 & 0 & 0.5376 & 0.3481 \\ 0 & 0.4524 & 0.4568 & 0.1942 & 0.3481 & 0 \end{bmatrix}$$

Open questions: Frequency invariant phase shift?

Constant phase at $\omega = \infty$

- $S_{22}(\infty) = \left.\frac{p}{q}\right|_{\omega=\infty} = e^{2.8607j}$.
- Necessary a constant phase shift in the whole frequency range.

Possible implementation

- A transmission line would work for narrow band.
- Possibility to include the dispersive behavior in the optimization.

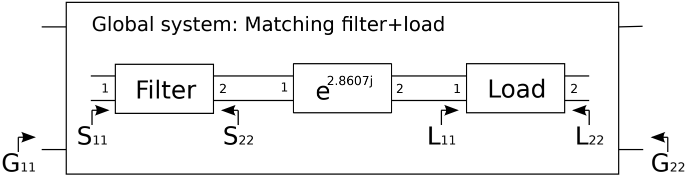


Figure 16: Global system with a phase shifter element.

Open question: Loss level for the matching filter?

Consideration for the losses.

- Need to consider the losses in the response of the matching filter.
- Improved reflection level: $\approx 8dB$ (from -2.104 to -10.27 dB).
- Maximum acceptable losses per resonator: 2 dB (4 resonators).

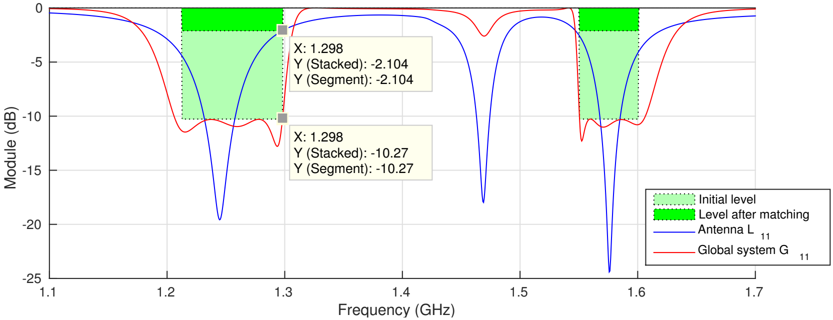


Figure 17: Improved band-reflection with the matching filter.

Perspectives

Apply the big problem to a generic load

This problem can be generalized as an interpolation problem, by imposing the value and the derivative of G_{22} equal to the ones of L_{22} in each of the transmission zeros of L_{21}.

Use of the transmission zeros for matching

When the load is of higher degree, we can use the transmission zeros as extra parameters to increase the number of matching points. However the requirements for the load in order to do so are still unknown.

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